

VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD

# B.E. (Mech. Engg.) IV Year I-Semester Main Examinations, December-2017 

# Finite Element Analysis 

Time: 3 hours

Note: Answer ALL questions in Part-A and any FIVE from Part-B

Part-A ( $10 \times 2=20$ Marks)

1. State stress equilibrium equations for $3 D$ Elastic body when it subjected to body and traction forces. Use usual symbols.
2. Estimate the values of shape function $\mathbf{N}_{1}$ and $\mathbf{N}_{2}$ for a bar element at one forth distance from both nodes.
3. A truss element is located in $X-Y$ coordinate system. The coordinates of its nodes are $(1,2)$ and $(-2,3)$ units. Deduce its transformation matrix.
4. The stiffness matrix of a plane frame is given as:

$$
10^{7}\left[\begin{array}{cccccc}
300 & 0 & 0 & -300 & 0 & 0 \\
0 & 0.75 & 1.5 & 0 & -0.75 & 1.5 \\
0 & 1.5 & 4 & 0 & 1.5 & 2 \\
-300 & 0 & 0 & 300 & 0 & 0 \\
0 & -.75 & -1.5 & 0 & 0.75 & -1.5 \\
0 & 1.5 & 2 & 0 & -1.5 & 4
\end{array}\right]
$$

Separate out the axial and bending stiffness matrix of the frame element.
5. Write down applications of Axisymmetric element and its advantages.
6. The plane area of three regions of triangular element are shown below: What are the shape functions $\mathbf{N}_{1}, \mathbf{N}_{\mathbf{2}}$ and $\mathbf{N}_{3}$ at interior point 4 .

7. Express the convective heat transfer matrix $\left[\mathrm{h}_{\mathrm{T}}\right]$ for a linear square cross section fin element of side 1 cm . Length of fin is 10 cm . and convective heat transfer coefficient: $50 \mathrm{~W} / \mathrm{cm}^{2}-{ }^{\circ} \mathrm{C}$.
8. The numerical integration of a function $\mathrm{F}(\mathrm{x})$ from -1 to +1 with the two points Gauss quadrature is given by: $\mathbf{F}(\mathbf{x})=W_{1} \mathbf{F}\left(\mathbf{x}_{1}\right)+W_{2} F\left(\mathbf{x}_{2}\right)$.
Express the values of $\mathbf{W}_{1}, \mathbf{x}_{1}, \mathbf{W}_{2}$ and $\mathbf{x}_{\mathbf{2}}$.
9. The consistent mass matrix of a bar element is given below: Convert it in to lumped mass matrix. $[\mathrm{M}]=10^{-2}\left[\begin{array}{ll}9.33 & 4.66 \\ 4.66 & 9.33\end{array}\right]$
10. Write down the polynomial required for a four node bar element to satisfy convergence criteria.

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\text { Part-B }(5 \times 10=50 \text { Marks })
$$

11. a) A uniform circular cross section link rotating at $\mathbf{2 0 0 0}$ r.p.m. in a vertical plane about one of the ends. Take density: $7.1 \mathrm{~g} / \mathrm{cc}$, Youngs modulus $=\mathbf{2 0 0 G P a}$, Length: $\mathbf{1 m}$, diameter: 1 cm . Discretize it in to two linear bar elements and consider only centrifugal force acting on it, Compute nodal displacements and elemental stresses.
b) State principle of minimum potential energy for the formulation element equation.
12. a) Discuss the behavior of Hermit shape functions for a beam element.
b) A simply supported rectangular cross section beam with point load is shown below: Take $\mathbf{E}=\mathbf{2 0 7} \mathbf{G P a}, \mathbf{b}=\mathbf{2 5 m m}$ and $\mathbf{b}=\mathbf{5 0} \mathbf{m m}$. With finite element approach, determine central deflection and rotation.

13. a) Why linear triangular element is called CST element?
b) Consider a portion of finite element model a plate of thickness $\mathbf{1 0} \mathbf{~ m m}$ as shown. A uniform traction force $\mathbf{T}$ acting along edges 7-8 and 8-9 of the plate. Determine the equivalent nodal forces at nodes 7,8 and 9 . All dimensions are in mm .

14. a) Why numerical integrations are employed in FEM?
b) With two linear elements approach, work out the temperature distribution in a thin rectangular fin of 120 mm long, 160 mm wide and 12.5 mm thick. The side of fin $(160 \mathrm{~mm} \times 1.25 \mathrm{~mm})$ is inside the wall which fin is at $330^{\circ} \mathrm{C}$. The ambient air temperature is $30^{\circ} \mathrm{C}$. Assume thermal conductivity: $0.2 \mathrm{~W} / \mathrm{mm}^{-}{ }^{\circ} \mathrm{C}$ and coefficient of convective heat transfer: $\mathbf{2} \times 10^{-4}$ W/mm ${ }^{2}{ }^{\circ}{ }^{\circ} \mathrm{C}$.
15. a) The area of cross section of straight bar element of length $L_{0}$ is $A_{0}$. Take density of element is: $\mathbf{q}$. Derive the expression for consistent mass matrix.
b) Elaborate convergence requirements in FEM.
16. a) The $x$-coordinates of nodes of a quadratic bar element are : $x_{1}=0, x_{2}=2$ and $x_{3}=6$. If a axial load $\mathbf{P}=\mathbf{1 0 0} \mathrm{N}$ acts on the element at $\mathbf{x}=4$, determine the nodal load vector.
b) A truss element of length 1 m and inclination $30^{\circ} \mathrm{C}$ with horizontal has global nodal displacement vector $\mathbf{Q}=\left[\begin{array}{lll}1 & 1.5 & -0.75 \\ 1.25\end{array}\right]^{\mathrm{T}}$. Find local nodal displacement vector $\mathbf{q}$.
17. Answer any two of the following:
a) Sketch isoparametric four noded quadrilateral element and deduce the shape functions
b) All faces of a square slab $\left(\mathrm{K}=\mathbf{7 W} \mathbf{~} / \mathbf{m}-{ }^{\circ} \mathrm{C}\right.$, side: $\left.\mathbf{1 m}\right)$ are insulated. It has an internal heat generation $Q=100 \mathrm{~W} / \mathbf{m}^{\mathbf{3}}$ at its centre. Determine the temperatures of opposite faces of slab assuming the heat flow only in one direction. Use two linear elements.
c) Write short notes on FEM softwares.
